Partonic Orbital Angular Momentum And Lorentz Invariant Relations

Abha Rajan
University of Virginia

POETIC 7, Temple University
Nov 15, 2016

People Involved

- Simonetta Liuti, University of Virginia
- Michael Engelhardt, New Mexico State University
- Gary Goldstein, Tufts University
- Aurore Courtoy, CINVESTAV Mexico
- Osvaldo Gonzalez, Old Dominion University and Jlab
- Brandon Kriesten, University of Virginia

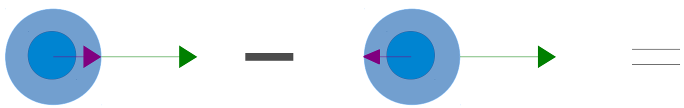
Rajan, Courtoy, Engelhardt and Liuti PRD 94 (2016)
Coutroy et al, Phys.Lett. B 731(2014)

Outline

- Spin Crisis !
- Orbital Angular Momentum
  - GTMD definition
  - GPD definition Ji
- What's the connection ? Lorentz Invariance and Equations of Motion
- Model calculations
- Final state interactions
- The parametrization used
- Conclusions

Proton Spin Crisis

$$\int_0^1 dx\, g_1^P(x) = \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right]$$



$$g_1^P(x)$$ Quark Spin Contribution

$$\frac{1}{2M}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}\left\langle P,S\left|\bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma_\mu\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right|P,S\right\rangle = \Lambda g_1(x)p_\mu + g_T(x)S_{\perp_\mu}$$

$1+\gamma^5$ ← Helicity Projection Operator

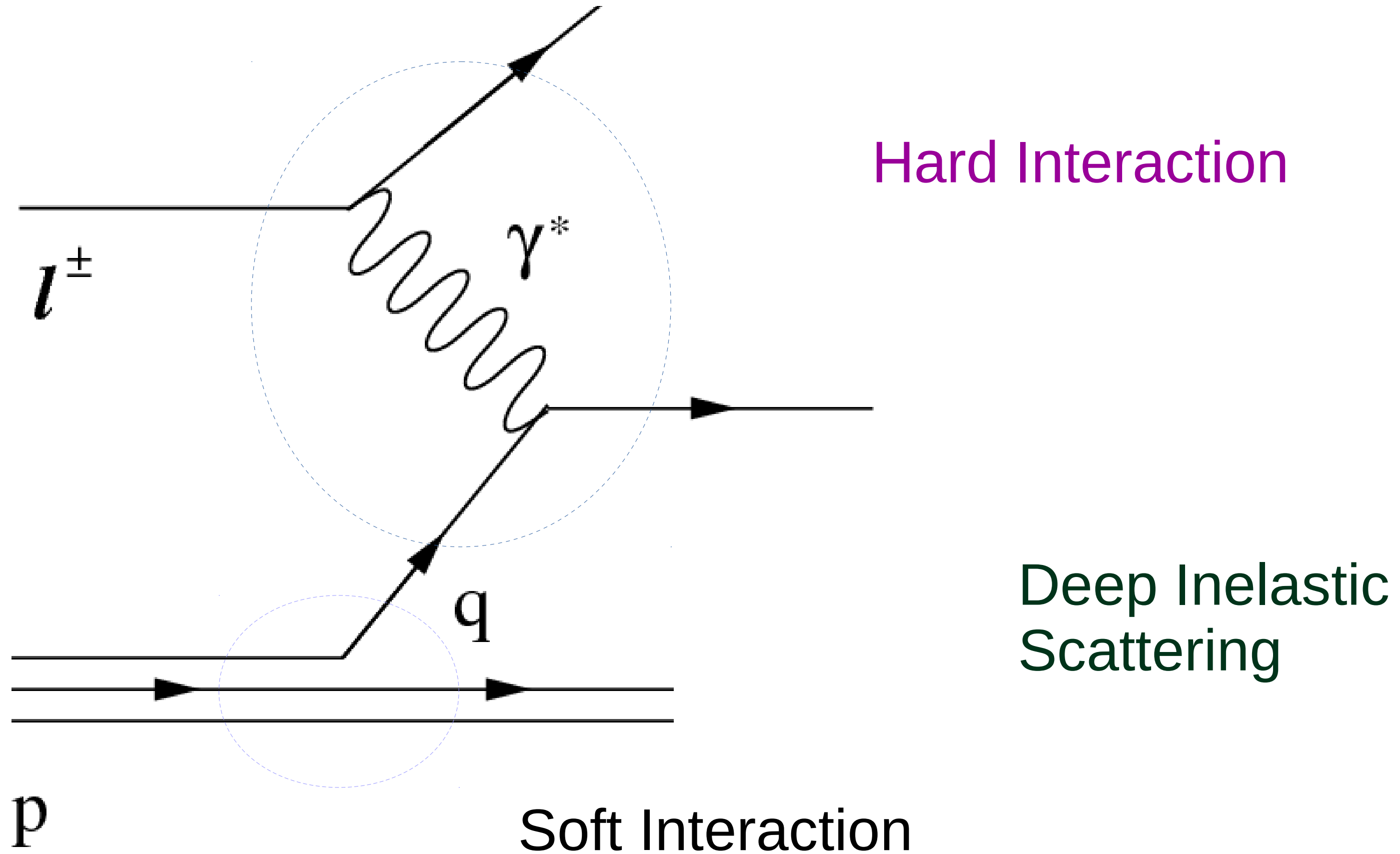
Measured by EMC experiment in 1980s to be only 33% of total !!

Spin Crisis !!!

Gluon Spin Contribution also small.

What are other sources ?
Orbital Angular Momentum

Hard and Soft Parts

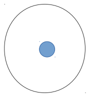


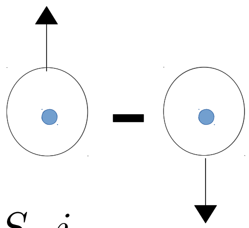
$$\int \frac{dz^-}{2\pi} e^{ik^+z^-} \langle p, S \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, S\rangle_{z^+=z_T=0} = f_1(x)$$

$$a^\pm = \frac{a^0 \pm a^3}{\sqrt{2}}$$

Transverse Momentum Distributions

- Transverse Momentum Distributions → include k_T transverse momentum of quarks





$$\int \frac{dz_- d^2 z_T}{2\pi} e^{ixP^+ z^- - k_T . z_T} \langle p, S \mid \bar{\psi}(-z/2)\gamma^+ \psi(z/2) \mid p, S\rangle_{z^+=0} = f_1(x, k_T) - \frac{\epsilon^{ij} k_T i S_T j}{M} f_{1T}^{\perp}(x, k_T)$$

Boglione, Mulders Phys Rev D60 (1999)

$$\int d^2 k_T f_1(x, k_T) = f_1(x)$$

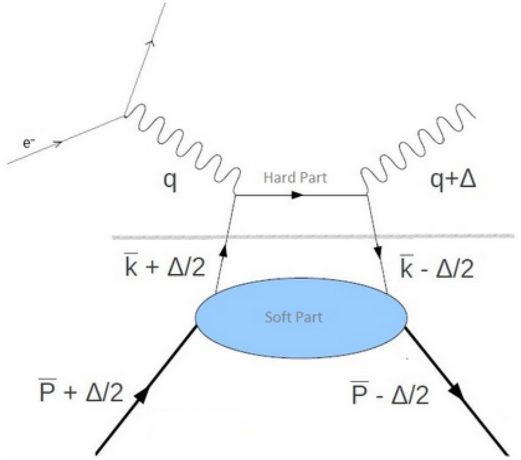
TMDs → Unintegrated PDFs

GPDs and GTMDs

- Generalized Parton Distributions : Off Forward PDFs

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0} = \bar{U}(P',\Lambda')(\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E(x,\xi,t))U(P,\Lambda)$$

$\xi = \frac{\Delta^+}{P^+}$ $\quad t = \Delta^2$ $\quad \Delta = P' - P$ Xiangdong Ji, PRL 78.610,1997



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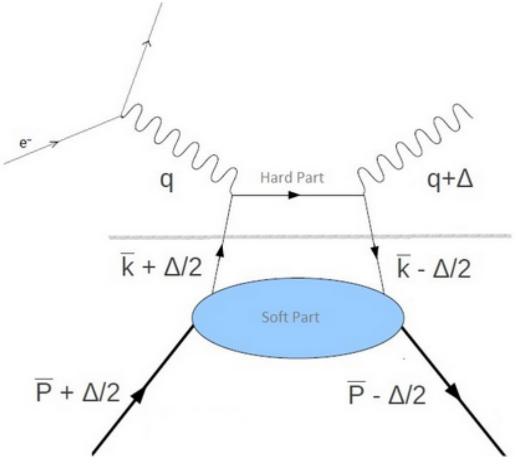

DVCS

- Enter at amplitude level

Ji Sum Rule : partonic angular momentum !

$$J_q = \frac{1}{2}\int_{-1}^{1} dx x (H_q(x,0,0) + E_q(x,0,0))$$

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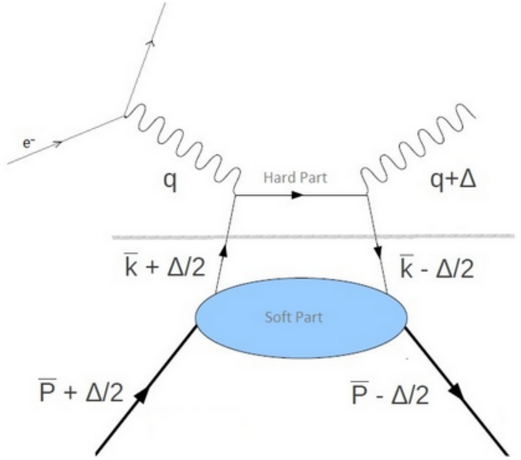

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- Generalised Transverse Momentum Distributions : Off forward TMDs

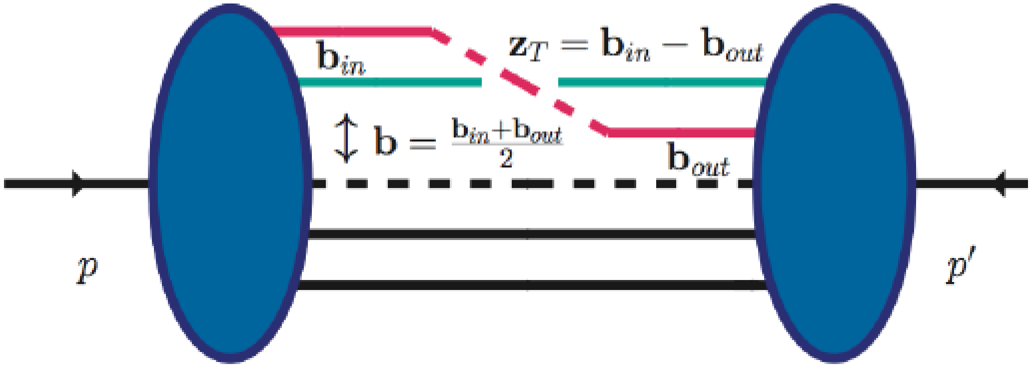
$$W^{\gamma^+}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{u}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]u(p,\Lambda)$$

Orbital Angular Momentum

Functions of $x, k_T^2, k_T.\Delta_T, \xi, t$

Meissner Metz and Schlegel, JHEP 0908 (2009)

A closer look at off forwardness



$$k \longleftrightarrow z$$

$$\Delta \longleftrightarrow b$$

Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

GPD based definition of Angular Momentum

$$\frac{1}{2} = J_q + J_g$$

$$\frac{1}{2}\Delta\Sigma + \mathcal{L}_q$$

Xiangdong Ji, PRL 78.610,1997

$$\vec{J}_q = \int d^3x \psi^\dagger [\vec{\gamma}\gamma^5 + \vec{x} \times (-i\vec{D})]\psi$$

$$J_q = \frac{1}{2}\int_{-1}^{1} dx x(H_q(x,0,0) + E_q(x,0,0))$$

GPDs : measured in exclusive experiments such as [deeply virtual compton scattering]

To access OAM, we take the difference between total angular momentum and spin

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

Direct description of OAM

$$\int dx\, x G_2 = \int dx\, x(H+E) - \int dx \tilde{H}$$

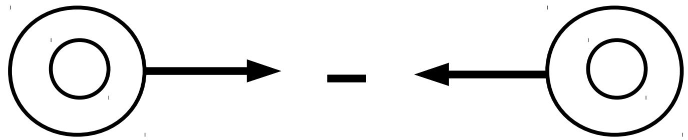
$$G_2 \equiv \tilde{E}_{2T} + H + E$$

Kiptily and Polyakov, Eur Phys J C 37 (2004)
Hatta and Yoshida, JHEP (1210), 2012

- The moment in x of the GPD G2 shown to be OAM
- Does not give us the distribution function for OAM

GTMD based definition

- How does F_{14} connect to OAM ?



Unpolarized quark in a longitudinally polarized proton

$$\mathcal{W}(x,\mathbf{k}_T,\mathbf{b}) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{ib.\Delta_T} \left[W^{\gamma^+}_{++} - W^{\gamma^+}_{--} \right]$$

$$W^{\gamma^+}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{u}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k^i_T}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta^i_T}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k^i_T\Delta^j_T}{M^2}F_{14}]u(p,\Lambda)$$

$$L = \int dx \int d^2k_T \int d^2\mathbf{b}(\mathbf{b}\times\mathbf{k}_T)\mathcal{W}(x,\mathbf{k}_T,\mathbf{b}) = -\int dx \int d^2k_T \frac{k_T^2}{M^2}F_{14}$$

Lorce et al PRD84, (2011)

The Two Definitions

- Weighted average of b_T X k_T
- Difference of total angular momentum and spin

$$\mathcal{L} = J - \frac{1}{2}\Delta\Sigma$$

The Two Definitions

- Weighted average of b_T X k_T

GTMD

- Difference of total angular momentum and spin

$$\mathcal{L} = J - \frac{1}{2}\Delta\Sigma$$

The Two Definitions

- Weighted average of b_T X k_T

GTMD

- Difference of total angular momentum and spin

$$\mathcal{L} = J - \frac{1}{2}\Delta\Sigma$$

GPD

Is there a connection ?

- We find that

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right)$$

- Unlike the previously known result, this is a distribution of OAM in x.
- Derived for a straight gauge link.

Quark- Quark Correlator

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle$$

The As (GPCFs)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2 z_T}{2\pi} e^{ixP^+z^- - k_T.z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=0}$$

GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=z_T=0}$$

GPDs

Generalized Lorentz Invariance Relations

- Parametrization of the quark quark correlator at different levels
- LIRs occur because the number of GPCFs is smaller than the number of GTMDs.

$$\begin{aligned}
\mathcal{W}^{[\gamma^\mu]}_{\Lambda\Lambda'} &= \frac{\bar{U}U}{M}(P^\mu A_1^F + k^\mu A_2^F + \Delta^\mu A_3^F) + i\frac{\bar{U}\sigma^{\mu k}U}{M}A_5^F + i\frac{\bar{U}\sigma^{\mu\Delta}U}{M}A_6^F \\
&+ i\frac{\bar{U}\sigma^{k\Delta}U}{M^3}(P^\mu A_8^F + k^\mu A_9^F + \Delta^\mu A_{17}^F)
\end{aligned}$$

$$W^{[\gamma^+]}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

$$F^{[\gamma^i]}_{\Lambda,\Lambda'} = \frac{1}{2(P^+)^2}\bar{U}\left[i\sigma^{+i}H_{2T} + \frac{\gamma^+\Delta_T^i}{2M}E_{2T} + \frac{P^+\Delta_T^i}{M^2}\tilde{H}_{2T} - \frac{P^+\gamma^i}{M}\tilde{E}_{2T}\right]U$$

Generalized Lorentz Invariance Relations

- The As are a function of the following scalar variables :

$$\sigma \equiv \frac{2k.P}{M^2}, \qquad \tau \equiv \frac{k^2}{M^2}, \qquad \sigma' \equiv \frac{k.\Delta}{\Delta^2} = \frac{k_T.\Delta_T}{\Delta_T^2}$$

For $\Delta^+ = 0$

$$\begin{aligned}\int dk^- A(k^2, k.P, k.\Delta \ldots) \quad &\rightarrow \quad \frac{M^2}{2P^+} \int d\sigma A \\ &\rightarrow \quad \frac{M^2}{2P^+} \int d\sigma' d\sigma d\tau \delta \left(\frac{k_T^2}{M^2} - x\sigma + \tau + \frac{x^2 P^2}{M^2} \right) \delta \left(\sigma' - \frac{k_T.\Delta_T}{\Delta_T^2} \right) A(\sigma, \tau, \sigma')\end{aligned}$$

Generalized Lorentz Invariance Relations

$$F_{14}^{(1)} = \int d\sigma d\sigma' d\tau \frac{M^3}{2} J \left[A_8^F + x A_9^F \right] \qquad J = \sqrt{x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}}$$

$$\tilde{E}_{2T} = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \left[\left(x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2} \right) A_9^F - \sigma' A_5^F - A_6^F \right]$$

$$H + E = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \sigma' A_5^F + A_6^F + \left(\frac{\sigma}{2} - \frac{x P^2}{M^2} \right) \left(A_8^F + x A_9^F \right)$$

$$\begin{aligned} -\frac{dF_{14}^{(1)}}{dx} &= \tilde{E}_{2T} + H + E \\ F_{14}^{(1)}(x) &= \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right) \end{aligned}$$

Equations of Motion

$$\int \frac{d^4z}{(2\pi)^4} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)(i\not{D} - m) i\sigma^{i+}\gamma_5 \psi(z/2) \mid p, \Lambda\rangle = 0$$

Some GTMDs occur with an explicit k_T coefficient, which along with the derivative leads to a k_T^2 moment leading to $F^{(1)}(x)$ type structure.

$$(\Lambda\Lambda') \rightarrow (++) - (--)$$

$$-F_{14}^{(1)}(x) = x\tilde{E}_{2T} - \tilde{H} + G^{(3)}$$

Use this and the LIR to derive Wandzura Wilczek Relations :

$$x(\tilde{E}_{2T} + H + E) = x\left[(H+E) - \int_x^1 \frac{dy}{y}(H+E) - \frac{1}{x}\tilde{H} + \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + G^{(3)}$$

$$\int dx(\tilde{E}_{2T} + H + E) = 0$$

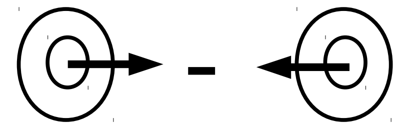
More LIRs

Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}'_{2T} + E'_{2T}\right) - \tilde{H}$$

$$\frac{dG_{12}^{e(1)}}{dx} = H'_{2T} - \frac{\Delta_T^2}{4M^2}E'_{2T} - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

$$\frac{dg_{1T}^{(1)}}{dx} = g_T + g_1$$



G_{11} describes a longitudinally polarized quark in an unpolarized proton. Measures spin orbit correlation.

Vector

$$\frac{dF_{12}^{e(1)}}{dx} = H_{2T}$$

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$

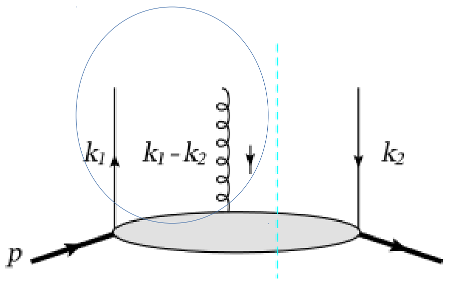
The GTMDs are complex in general. $F_{12}^{o} \rightarrow f_{1T}^{\perp}$

$$X = X^e + iX^o$$

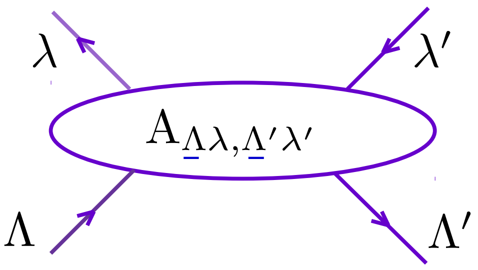
The imaginary part integrates to zero, on integration over k_T.

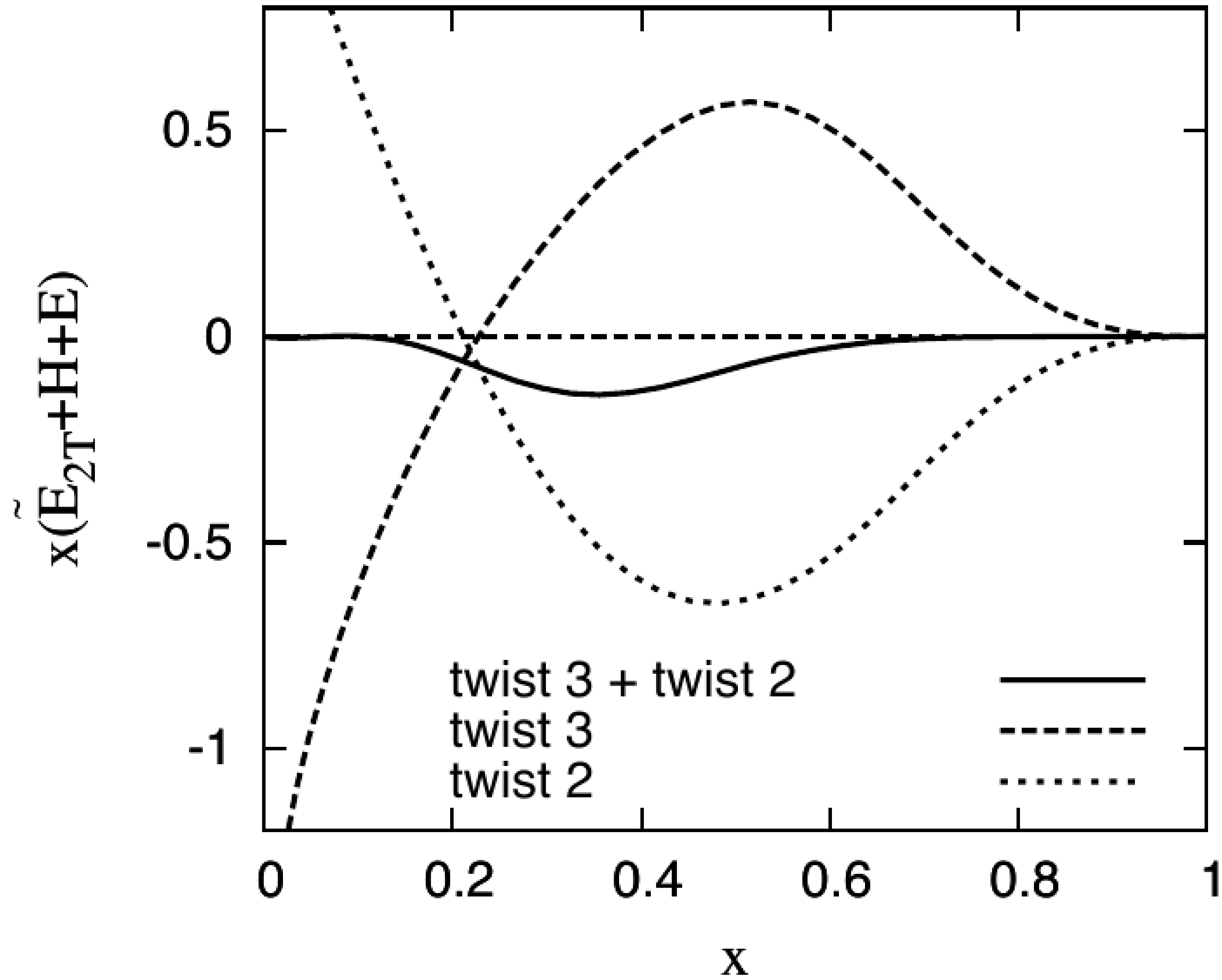
Model Calculations

- Diquark model at twist three
- Use projection into good and bad components to form helicity amplitudes
- Bad component is a composite quark gluon structure



$$\bar{\psi}\gamma^1\psi = \chi_R^*\phi_R - \chi_L^*\phi_L - \phi_L^*\chi_L + \phi_R^*\chi_R$$





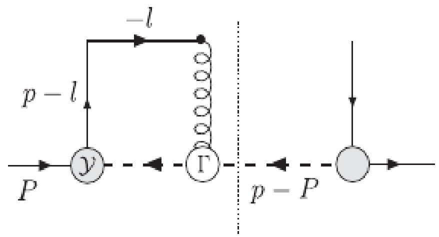
Rajan, Courtoy, Engelhardt and Liuti, PRD94, 2016

Including Final State Interactions

Talk by Brandon Kriesten

- Ji → Straight Gauge link

- Jaffe Manohar → Staple Link



- The difference is the torque

$$L_q^{JM} - L_q^{Ji} = \int \frac{d^2 z_T dz^-}{(2\pi)^3} \langle P', \Lambda' | \bar{\psi}(z) \gamma^+ (-g) \int_{z^-}^{\infty} dy^- U \left[z_1 G^{+1}(y^-) - z_2 G^{+2}(y^-) \right] U \psi(z) | P, \Lambda \rangle \Big|_{z^+=0}$$

Burkardt (2013)

Including Final State Interactions

- The twist three GPDs are obtained by integrating over the gluon momentum for a quark quark gluon quark operator

$$g_T(x) = \frac{1}{2x} \int \mathrm{d}y \left[\tilde{G}(x,y) + \tilde{G}(y,x) + G(x,y) - G(y,x) \right]$$

Jaffe and Ji, 1992

- Final state interactions on the other hand connect to a gluonic pole of Qiu Sterman like term

$$T_{q,F}(x,x) = \frac{1}{M} \int d^2k_\perp k_\perp^2 f_{1T}^\perp(x, k_\perp^2)$$

Kang, Qiu and Zhang PRD 81, 2010

- Hence in some sense different limits of the same correlator

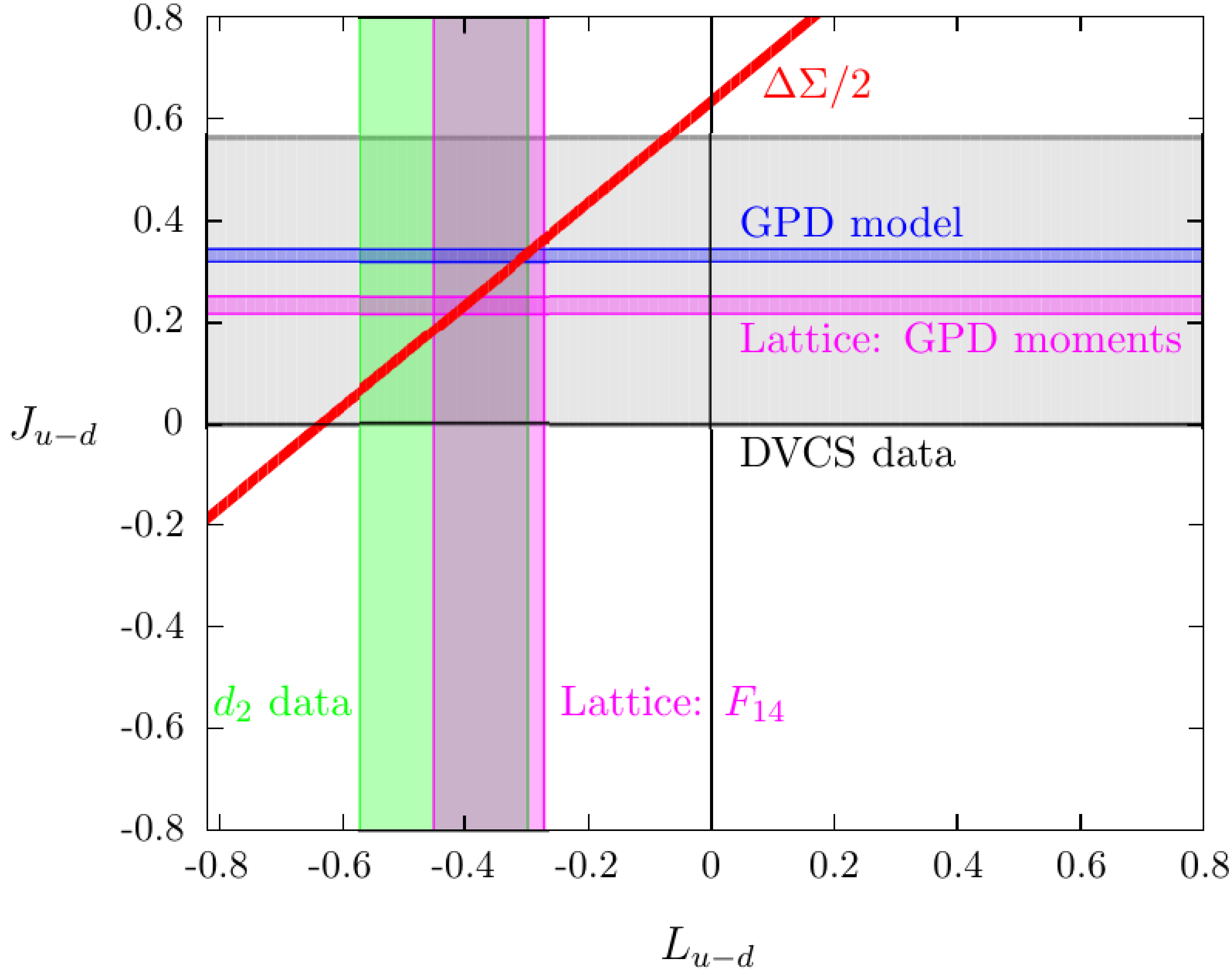
The Parametrization and the gauge link structure

$$W^{\gamma^+}_{\Lambda\Lambda'} = \int \frac{dz_- d^2 z_T}{2\pi} e^{ixP^+ z^- - k_T . z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\gamma^+ U(-z/2, z/2|n)\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

- Without the gauge link, the number of As matches that of the GPDs.
- If we include the gauge link, new As are introduced and the number then matches the number of GTMDs.
- Hence, LIRs need not exist anymore, the new terms also called 'LIR breaking' terms.

Conclusions

- We have shown a way to connect GPDs and GTMDs.
- Hence there is a way to measure effects that were solely associated with GTMDs by measuring the associated GPD.
- Quark gluon quark interactions are at the heart of twist three effects.
- Highlights the role of genuine twist twist three contributions and 'wandzura wilczek' terms.
- A way to include final state interactions. Interesting how the intrinsic transverse momentum at twist two connects to gluon effects at twist three.



0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
J_{u-d}
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
L_{u-d}
$\Delta\Sigma/2$
GPD model
Lattice: GPD moments
DVCS data
d_2 data
Lattice: F_{14}